Analytical Solutions for Thermal Stresses in Buried Channel Waveguides

M. Huang

Abstract—Thermal stresses are important factors in determining the birefringence of optical waveguides. Analytical solutions are developed to estimate the thermal stresses in the core of a buried rectangular channel waveguide. The closed-form solutions for various situations are obtained. The results show that the thermal expansion mismatch between the upper cladding and substrate plays significant role in determining the anisotropy of the thermal stresses in the core. The analytical solutions developed in this paper are consistent with finite element simulations and experimental results.

Index Terms—Birefringence, optical polarization, optical waveguides, stress.

I. INTRODUCTION

THERMAL expansion mismatch among substrate cladding layers and core can induce thermal stresses in optical waveguides. Due to the elasto-optic effect, these stresses can cause the birefringence. Many methods have been proposed to control the optical performance by tuning the stresses in the optical devices. These include: Si-birefringence compensation film [1], stress-applying silicon film [2], silica substrate [3], stress release groove [4], metal plate [5], and tuning the thermal expansion coefficient of the upper cladding layer [6]–[8]. These methods demonstrate the importance of studying *device mechanics*, an interdisciplinary field to investigate the stress/strain effects on the performance of solid-state devices and engineer the stress/strain field to improve the device performance. A powerful way to estimate the stress distribution is the numerical method, such as finite element simulation, but analytical solutions are very useful in developing the intuition to understand the thermal stresses in the waveguides and convenient to be used in the preliminary waveguide design. The author developed a method to estimate the average thermal stresses in the core of a buried rectangular channel waveguide in [9]. This paper gives a detailed description of this method, provides modified closed-form solutions, and studies the validation range of the solutions. The results predicted by the analytical solutions are consistent with the finite element simulations and experimental results from other works [6], [7].

II. THERMAL STRESSES IN THE WAVEGUIDE CORE

The structure of a buried rectangular channel waveguide is shown in Fig. 1(a). The mechanical and thermal properties of

Fig. 1. (a) Waveguide structure. (b) Finite element simulation of stress distributions in the core. In the simulation, the core has square shape. The materials properties used are $E_{\text{core}} = E_{\text{cladding}} = 70$ GPa, $\nu_{\text{core}} = \nu_{\text{cladding}} = 0.2$, $\alpha_{\text{core}} = 1.2 \times 10^{-6}/\text{K}$, $\alpha_{\text{cladding}} = 3.0 \times 10^{-6}/\text{K}$, $\alpha_{\text{substrate}} = 3.6 \times 10^{-6}/\text{K}$, and the temperature difference from the stress-free state is $\Delta T = -1000$ K.

the upper cladding, lower cladding, core and substrate are different. When the temperature changes, the thermal mismatch will induce thermal stresses in the core. Fig. 1(b) shows the thermal mismatch induced σ_{xx} and σ_{yy} distributions in the core. They are obtained by the finite element simulation using the commercial software, ABAQUS (version 6.3, Hibbitt, Karlsson & Sorensen, Inc., Pawtucket, RI, 2002). We can see that the thermal stresses in the core are inhomogeneous and anisotropic. Stress nonuniformity seems to have little effects on the birefringence of a slab waveguide [10], but may have some effects on the birefringence of an embedded channel waveguide with small and high index core [7]. As a first estimation, the stress nonuniformity in the core is ignored in this paper. The closed-form solutions to estimate the average thermal stresses and stress anisotropy in the core are developed.

Due to the thermal expansion mismatch between the substrate and cladding, the biaxial stress will be induced in the cladding layers far away from the core. Because the substrate is much

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The author is with the Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544 USA (e-mail: min-huang@alumni.princeton.edu).

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thicker than the cladding layers, the biaxial stress in the cladding layers far away from the core can be expressed as [9]

$$\sigma_{uc} = E_{uc}^*(\alpha_s - \alpha_{uc})\Delta T \quad (1)$$

$$\sigma_{lc} = E_{lc}^*(\alpha_s - \alpha_{lc})\Delta T \quad (2)$$

where $E^* = E/(1 - \nu)$ is defined as biaxial modulus, and E , ν , and α are Young's modulus, Poisson's ratio, and coefficient of thermal expansion (CTE), respectively. The subscripts s , uc , and lc refer to the substrate, upper cladding and lower cladding, respectively. ΔT is the temperature change. It is important to note that the biaxial stresses given by (1) and (2) are only valid when the cladding is far away from the core and in the plane parallel to the substrate. The stresses in and around the core are not biaxial, which will be studied in detail.

Due to the thermal expansion mismatch among core, lower cladding, upper cladding and substrate, the thermal stresses are induced in the core. Because the waveguide is very thin compared to the substrate, the strain in the z direction is the same as that of the substrate, i.e., $\varepsilon_{zz}^{\text{core}} = \varepsilon_{zz}^{uc} = \alpha_s \Delta T$. This is a generalized plane strain problem. The stresses in the x , y , and z directions are inhomogenous, i.e., $\sigma_{xx}^{\text{core}}$, $\sigma_{yy}^{\text{core}}$, and $\sigma_{zz}^{\text{core}}$ are not uniformly distributed in the core, as shown in Fig. 1(b). To develop closed-form solutions, it is assumed in this paper that the thermal stresses in the core are uniformly distributed. We will focus on the anisotropy of the average thermal stresses, which is directly related to birefringence.

Free body diagram is used to analyze the stress state in core and cladding. The core is considered to be separated from the waveguide, and is applied with homogenous stresses, $\sigma_{xx}^{\text{core}}$, $\sigma_{yy}^{\text{core}}$, and $\sigma_{zz}^{\text{core}}$, along x , y , and z directions. The stress state in the core is shown in Fig. 2(a). When the core is separated from the waveguide, a rectangular hole is left in the upper cladding and the stresses are applied on the surfaces of the hole, as shown in Fig. 2(b). The stresses applied on the surfaces of the hole should be equal to $\sigma_{xx}^{\text{core}}$ and $\sigma_{yy}^{\text{core}}$ due to Newton's third law of motion. Another remote stress, $\sigma_{uc} = E_{uc}(\alpha_s - \alpha_{uc})\Delta T/(1 - \nu_{uc})$, is applied on the upper cladding far away from the core due to the constraint from the substrate. The stress in the y direction in the upper cladding beside the top side of the hole and the stress in the x direction in the upper cladding beside the right side of the hole can be superimposed by the stress concentration of $\sigma_{uc} = E_{uc}(\alpha_s - \alpha_{uc})\Delta T/(1 - \nu_{uc})$, $\sigma_{xx}^{\text{core}}$ and $\sigma_{yy}^{\text{core}}$, as shown in Fig. 2(b). The methods to calculate the stress concentration are shown in Fig. 3. k_{ij} is the stress concentration factor along a side of the rectangular hole. The first subscript is the direction to apply remote stress, and the second subscript is the stress

direction. For example, when the remote stress σ is applied along y direction, the tensile stress $k_{yy}\sigma$ is concentrated at the top (or bottom) side of the hole, and the compressive stress $-k_{yx}\sigma$ is concentrated at the right (or left) side of the hole. The stress concentration factors are determined by the aspect ratio of the core, the spacing between the cores, and the thickness of the upper cladding. The effects will be studied later.

The core and upper cladding are assumed to be isotropic materials, so the strains in the y direction beside the top interface between core and upper cladding are

core:

$$\varepsilon_{yy}^{\text{core}} = \frac{\sigma_{yy}^{\text{core}}}{E'_{\text{core}}} - \nu'_{\text{core}} \frac{\sigma_{xx}^{\text{core}}}{E'_{\text{core}}} + \alpha'_{\text{core}} \Delta T \quad (3)$$

upper cladding:

$$\varepsilon_{yy}^{uc} = \frac{k_{xy}\sigma_{xx}^{\text{core}} + k_{yy}\frac{E_{uc}}{1-\nu_{uc}}(\alpha_s - \alpha_{uc})\Delta T - (k_{yy} - 1)\sigma_{yy}^{\text{core}}}{E'_{uc}} - \nu'_{uc} \frac{\sigma_{xx}^{\text{core}}}{E'_{uc}} + \alpha'_{uc} \Delta T. \quad (4)$$

And the strains in the x direction beside the side interface between core and upper cladding are

core:

$$\varepsilon_{xx}^{\text{core}} = \frac{\sigma_{xx}^{\text{core}}}{E'_{\text{core}}} - \nu'_{\text{core}} \frac{\sigma_{yy}^{\text{core}}}{E'_{\text{core}}} + \alpha'_{\text{core}} \Delta T \quad (5)$$

upper cladding:

$$\varepsilon_{xx}^{uc} = \frac{k_{yx}\sigma_{yy}^{\text{core}} - k_{yx}\frac{E_{uc}}{1-\nu_{uc}}(\alpha_s - \alpha_{uc})\Delta T - (k_{xx} - 1)\sigma_{xx}^{\text{core}}}{E'_{uc}} - \nu'_{uc} \frac{\sigma_{yy}^{\text{core}}}{E'_{uc}} + \alpha'_{uc} \Delta T \quad (6)$$

where $E' = E/(1 - \nu^2)$, $\nu' = \nu/(1 - \nu)$ and $\alpha' = (1 + \nu)\alpha - \nu\alpha_s$ are, respectively, the plane strain Young's modulus, Poisson's ratio, and thermal expansion coefficient.

Because there is no debonding between the core and the cladding, the strain in the y direction should be continuous across the top interface, and the strain in the x direction should be continuous across the side interface, i.e.,

$$\text{Top interface: } \varepsilon_{yy}^{\text{core}} = \varepsilon_{yy}^{uc} \quad (7)$$

$$\text{Side interface: } \varepsilon_{xx}^{\text{core}} = \varepsilon_{xx}^{uc} \quad (8)$$

Substituting (3)–(6) into (7) and (8) gives the thermal stresses in the x and y directions in the core, as shown in (9) and (10) at the bottom of the page. The core stress in the z direction can be obtained by the generalized plane strain condition as

$$\sigma_{zz}^{\text{core}} = E_{\text{core}}(\alpha_s - \alpha_{\text{core}})\Delta T + \nu_{\text{core}}(\sigma_{xx}^{\text{core}} + \sigma_{yy}^{\text{core}}). \quad (11)$$

$$\sigma_{xx}^{\text{core}} = \frac{\left(\frac{1}{E'_{\text{core}}} + \frac{k_{yy}-1}{E'_{uc}}\right)[\alpha'_{uc} - \alpha'_{\text{core}} - k_{yx}(\alpha_s - \alpha'_{uc})] + \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{yx}-\nu'_{uc}}{E'_{uc}}\right)[\alpha'_{uc} - \alpha'_{\text{core}} + k_{yy}(\alpha_s - \alpha'_{uc})]}{\left(\frac{1}{E'_{\text{core}}} + \frac{k_{yy}-1}{E'_{uc}}\right)\left(\frac{1}{E'_{\text{core}}} + \frac{k_{xx}-1}{E'_{uc}}\right) - \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{yx}-\nu'_{uc}}{E'_{uc}}\right)\left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{xy}-\nu'_{uc}}{E'_{uc}}\right)} \Delta T \quad (9)$$

$$\sigma_{yy}^{\text{core}} = \frac{\left(\frac{1}{E'_{\text{core}}} + \frac{k_{xx}-1}{E'_{uc}}\right)[\alpha'_{uc} - \alpha'_{\text{core}} + k_{yy}(\alpha_s - \alpha'_{uc})] + \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{xy}-\nu'_{uc}}{E'_{uc}}\right)[\alpha'_{uc} - \alpha'_{\text{core}} - k_{yx}(\alpha_s - \alpha'_{uc})]}{\left(\frac{1}{E'_{\text{core}}} + \frac{k_{yy}-1}{E'_{uc}}\right)\left(\frac{1}{E'_{\text{core}}} + \frac{k_{xx}-1}{E'_{uc}}\right) - \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{yx}-\nu'_{uc}}{E'_{uc}}\right)\left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{k_{xy}-\nu'_{uc}}{E'_{uc}}\right)} \Delta T \quad (10)$$

Fig. 2. Free body diagram of a buried channel waveguide. (a) Stresses in the core. (b) Stresses in the upper cladding beside the interface.

Equations (9)–(11) give the closed-form solutions to estimate the average stresses in the core. It is important to note that the core stress in the direction perpendicular to the substrate is not always zero, and the core stress in the y direction is not equal to that in the z direction. Therefore, the stress in the waveguide core is not equally biaxial, although the stress in the cladding far away from the core is equally biaxial [9].

For many waveguides, the core is nearly square shape, and the spacing between cores and the thickness of upper cladding layer are somewhat greater than the size of the core, then the stress concentration factor can be simplified as $k_{xx} = k_{yy} = k$ and $k_{xy} = k_{yx} = k'$. k and k' can be estimated as 3 and 1 by using finite element simulation. The core stresses in different directions for this kind of waveguide can be simplified from (9)

and (10) as shown in (12) and (13) at the bottom of the next page. Subtracting (12) from (13) leads to the stress anisotropy in the plane perpendicular to the light propagation direction as

$$\sigma_{yy}^{core} - \sigma_{xx}^{core} \approx \frac{4(1 + \nu_{uc})(\alpha_s - \alpha_{uc})\Delta T}{\frac{(1 + \nu_{core})}{E_{core}} + \frac{(1 + \nu_{uc})(3 - 4\nu_{uc})}{E_{uc}}}. \quad (14)$$

Equation (14) shows that for this kind of waveguide, the core stress anisotropy in the direction perpendicular to the light propagation direction is mainly determined by thermal mismatch between upper cladding layer and substrate $\alpha_s - \alpha_{uc}$ and is zero when $\alpha_s = \alpha_{uc}$. And the thermal expansion of lower cladding layer and core do not play important role in determining the anisotropy of the core stress. These are consistent with finite element simulation and experimental results [6]–[8].

Fig. 3. Stress concentration along a side of a rectangular hole.

The lower cladding is fully constrained by the substrate, so it can only transfer the effect of the substrate, and its properties do not significantly influence the deformation of the core. The core thermal stresses $\sigma_{xx}^{\text{core}}$ and $\sigma_{yy}^{\text{core}}$ are mainly determined by the thermal expansion of the core, the upper cladding and the substrate, but the thermal stress anisotropy between x and y directions $\sigma_{yy}^{\text{core}} - \sigma_{xx}^{\text{core}}$ is largely determined by the thermal expansion mismatch between the upper cladding and the substrate. The effects of the thermal expansion of the core are cancelled out!

In some waveguides, the upper cladding and the core are made from the same materials with different doping, so the Young's modulus and Poisson's ratio do not have much difference, i.e., $E_{\text{core}} \approx E_{\text{uc}} \approx E$ and $\nu_{\text{core}} \approx \nu_{\text{uc}} \approx \nu$. Under these conditions, (12)–(14) can be further simplified as

$$\sigma_{xx}^{\text{core}} \approx \frac{E(\alpha_{\text{uc}} - \alpha_{\text{c}})\Delta T}{2(1-\nu)} \quad (15)$$

$$\sigma_{yy}^{\text{core}} \approx \frac{E(\alpha_{\text{uc}} - \alpha_{\text{c}})\Delta T}{2(1-\nu)} + \frac{E(\alpha_{\text{s}} - \alpha_{\text{uc}})\Delta T}{1-\nu} \quad (16)$$

$$\sigma_{yy}^{\text{core}} - \sigma_{xx}^{\text{core}} \approx \frac{E(\alpha_{\text{s}} - \alpha_{\text{uc}})\Delta T}{1-\nu}. \quad (17)$$

In the solutions developed previously [9], k and k' are not distinguished, and the stress concentration factor used is $k - k' \approx 2$.

Equations (15) and (16) show that the simplifications in [9] do not change the results for this situation.

Equations (12)–(14) are valid when the core is nearly square shape, and the spacing between cores and the thickness of upper cladding layer are somewhat greater than the size of the core. To exploit the validation range of the solutions, one needs to study the effects of the aspect ratio of the core, the spacing between the cores, and the upper cladding thickness on the stress concentration factors. Finite element simulation software, ABAQUS, is used to calculate the stress concentration factors along the side of a rectangular hole under different aspect ratio of the core, spacing between the cores, and upper cladding thickness. The results are shown in Figs. 4–6. We can see that the stress concentration factors are sensitive to the aspect ratio of the core. When the spacing between the cores and the thickness of the upper cladding are larger than ~ 3 times of the core size, the effects of the interacting among different cores and the thickness of the upper cladding can be ignored. Most waveguides satisfy this condition. If the aspect ratio of the core is away from 1, or the spacing among the cores and the upper cladding are smaller than 3 times of the core size, the thermal stresses will be highly nonuniformly distributed in the core, which invalidates the assumption used in developing the analytical solutions. Even under this situation, the solutions developed in this paper may still provide some trends, but the detailed stress distribution needs to be obtained by numerical methods.

$$\sigma_{xx}^{\text{core}} \approx \frac{\left(\frac{1}{E'_{\text{core}}} + \frac{2}{E'_{\text{uc}}}\right)(2\alpha'_{\text{uc}} - \alpha'_{\text{core}} - \alpha_{\text{s}}) + \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{1-\nu'_{\text{uc}}}{E'_{\text{uc}}}\right)(3\alpha_{\text{s}} - \alpha'_{\text{core}} - 2\alpha'_{\text{uc}})}{\left(\frac{1}{E'_{\text{core}}} + \frac{2}{E'_{\text{uc}}}\right)^2 - \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{1-\nu'_{\text{uc}}}{E'_{\text{uc}}}\right)^2} \Delta T \quad (12)$$

$$\sigma_{yy}^{\text{core}} \approx \frac{\left(\frac{1}{E'_{\text{core}}} + \frac{2}{E'_{\text{uc}}}\right)(3\alpha_{\text{s}} - 2\alpha'_{\text{uc}} - \alpha'_{\text{core}}) + \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{1-\nu'_{\text{uc}}}{E'_{\text{uc}}}\right)(2\alpha'_{\text{uc}} - \alpha'_{\text{core}} - \alpha_{\text{s}})}{\left(\frac{1}{E'_{\text{core}}} + \frac{2}{E'_{\text{uc}}}\right)^2 - \left(\frac{\nu'_{\text{core}}}{E'_{\text{core}}} + \frac{1-\nu'_{\text{uc}}}{E'_{\text{uc}}}\right)^2} \Delta T \quad (13)$$

Fig. 4. Effects of aspect ratio on the stress concentration factors. The spacing between cores and the upper cladding thickness are much larger than the core size.

Fig. 5. Effects of the spacing between the cores on the stress concentration factors. The core has square shape and the upper cladding thickness is much larger than the core size.

Fig. 6. Effects of upper cladding thickness on the stress concentration factors along a side of the hole. The core has square shape and the spacing between the cores is much larger than the core size.

III. COMPARISON WITH FINITE ELEMENT SIMULATIONS AND EXPERIMENTAL RESULTS

To validate the closed-form analytical solutions obtained in this paper, a comparison with finite element simulation is made. In the finite element simulation, the waveguide is treated as a generalized plane strain problem, i.e., the strain along z direction is a constant. The calculation is done by using

Fig. 7. Effects of the thermal-expansion mismatch between the upper cladding and substrate on the thermal stress anisotropy in the core. The solid line is (14). The dots are the finite element simulations for different substrate CTEs.

the commercial finite element software, ABAQUS, adopting four-node quadrilateral plane strain node. In the simulation, the core is considered as square shape. The materials properties used are $E_{\text{core}} = E_{\text{uc}} = 70$ GPa, $\nu_{\text{core}} = \nu_{\text{uc}} = 0.2$, $\alpha_{\text{core}} = 1.2 \times 10^{-6}/\text{K}$, and the temperature difference from the stress-free state is $\Delta T = -1000$ K. Fig. 7 shows the effects of the CTE mismatch between the upper cladding and the substrate on the stress anisotropy in the core. The solid line is (14), and the dots are the stress anisotropy at the core center obtained by finite element simulation. We can see that the closed-form solution developed in this paper is consistent with the finite element simulation and the difference between σ_{xx} and σ_{yy} can be tuned by varying the thermal expansion mismatch between the upper cladding and the substrate. When the CTE mismatch between the upper cladding and the substrate is zero, the difference between σ_{xx} and σ_{yy} is also nearly zero.

Kilian *et al.* [7] also obtained the stresses in the core of the optical waveguide under various scenario by using finite element simulation. The comparison between the analytical solutions with their numerical results is listed in Table I. The analytical solutions are obtained by (11)–(13), except the evanescent field coupler case, where the effect of the smaller core spacing on the stress intensity factor is considered. From the finite element simulation results obtained by Kilian *et al.* [7], we can see that the substrate thickness t_s , upper cladding thickness t_{uc} , and lower cladding CTE, α_{lc} play little roles in determining the stresses in the core, and core CTE α_{core} , and upper cladding CTE α_{uc} , are important parameters in determining the stresses in the core. These predictions can be clearly shown in the analytical solutions developed in this paper [(12), (13)]. The quantitative comparison between the finite element simulation and the analytical solution shows reasonable consistency, except σ_{xx} in the evanescent field coupler case, where the small spacing between the cores induces large stress inhomogeneity.

As a first approximation, the stress induced polarization shift can be estimated by the thermal stresses in the core, although

TABLE I
COMPARISON OF THE ANALYTICAL SOLUTIONS DEVELOPED IN THIS PAPER AND FINITE ELEMENT SIMULATIONS IN [7]

Substrate	silica		Silicon												
$\alpha_{\text{core}} (10^{-6}/\text{K})$	1.0		1.8			1.8					1.8		2.4		
$\alpha_{\text{uc}} (10^{-6}/\text{K})$	1.2		2.4			3.45					3.45		3.45		
$t_s (\text{mm})$	analytical solution	1	analytical solution	1	1	analytical solution	1	0.5	1	1 [*]	analytical solution	1 ^{**}	analytical solution	1	1
$t_{\text{uc}} (\mu\text{m})$		30		30	30		30	30	20	30 [*]		30 ^{**}		30	30
$\alpha_{\text{lc}} (10^{-6}/\text{K})$		0.8		1.2	1.8		1.2	1.2	1.2	1.2 [*]		1.2 ^{**}		1.2	1.8
$\sigma_{xx} (\text{MPa})$	-8.27	-8.99	-25.3	-26.16	-26.16	-68.9	-71.57	-73.72	-70.26	-71.57 [*]	-33.2	-67.47 ^{**}	-43.9	-45.5	-45.54
$\sigma_{yy} (\text{MPa})$	25.0	20.33	-125	-105.4	-109.4	-81.4	-71.8	-68.83	-72.15	-71.8 [*]	-57.4	-48.13 ^{**}	-56.3	-45.8	-48.6
$\sigma_{zz} (\text{MPa})$	17.3	10.6	-156	-130.9	-135.6	-156	-142.4	-140.9	-144.7	-142.4 [*]	-144	-137.6 ^{**}	-104	-89.5	-93.0

* Diffusion: Upper 5 μm of cladding are assumed to have α of $0.9 \times 10^{-6} \text{ 1/K}$ to examine the influence of out-diffusion of dopants.

** Evanescent Field Coupler: two 6 by 6 μm core ridges spaced 6 μm apart homogeneous cladding.

Fig. 8. Effects of the thermal-expansion coefficients of the upper cladding and the substrate on the polarization shift. The solid line is (18), and the dots are the experimental results from [6].

in most practical single mode waveguides the optical fields extend considerably into the claddings. The solutions based on effective index method and linear approximation to estimate the anisotropic stress induced polarization shift has been developed in [5]. The stress induced polarization shift can be estimated as [5]

$$\lambda_{\text{TM}} - \lambda_{\text{TE}} \approx \lambda \frac{C_2 - C_1}{n_0} \frac{n_h^2}{n_{\text{eff}}^2} A_w \times \left[(\sigma_{xx} - \sigma_{zz}) - \frac{n_0^2}{n_h^2} A_t (\sigma_{yy} - \sigma_{zz}) \right] \quad (18)$$

where n_{eff} and n_0 are, respectively, the effective index and the refractive index of the core under stress-free state, and n_h is the TM effective index of the equivalent slab waveguide. C_1 and C_2 are stress-optic constants. A_w and A_t are coefficients defined in [5]. Fig. 8 shows the effects of the thermal expansion mismatch between the upper cladding and the substrate on the polarization shift. The solid line is (18), and the dots are the experimental results from [6]. In the calculation, the size of the waveguide is taken to be $6 \mu\text{m} \times 6 \mu\text{m}$, and wavelength is taken to be $1.55 \mu\text{m}$. We can see that the analytical prediction based on effective index method is qualitatively consistent with the experimental results from [6]. The results suggest that by varying the

thermal expansion coefficient of the upper cladding, the polarization shift can be tuned over a wide range. This can be used to control the polarization shift because the thermal expansion coefficient of upper cladding glass can be tuned over a wide range [11].

IV. CONCLUDING REMARKS

This paper analytically studied thermal stresses in the optical waveguides. The closed-form approximate solutions developed in this paper are consistent with the finite element simulations and experimental results. The stresses considered in this work are merely induced by the thermal expansion mismatch among dissimilar materials, and the shape of the core is rectangular. It is important to note that the polarization shift can be reduced two-fold just by changing the waveguide core to a pedestal structure [8], or can be varied by changing fabrication process [12]. This warrants closer attention in further studies.

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M. Huang received the B.S.E. and M.S.E. degrees in engineering mechanics from Xi'an Jiaotong University, Xi'an, China, in 1992 and 1995, respectively, and the Ph.D. degree in mechanical engineering from Princeton University, Princeton, NJ, in 2001.

He is currently a Lecturer and Research Associate in the Department of Mechanical and Aerospace Engineering, Princeton University. His present research interests focus on the mechanics and materials in electronic, photonic, and biological micro/nano systems. He has published over 35 papers in technical journals and holds one U.S. patent.